Dark matter and the first stars: a new phase of stellar evolution

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A mechanism is identified whereby dark matter (DM) in protostellar halos dramatically alters the current theoretical framework for the formation of the first stars. Heat from neutralino DM annihilation is shown to overwhelm any cooling mechanism, consequently impeding the star formation process and possibly leading to a new stellar phase. A "dark star" may result: a giant ($\gtrsim$ 1 AU) hydrogen-helium star powered by DM annihilation instead of nuclear fusion. Observational consequences are discussed.

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The first stars in the universe mark the end of the cosmic dark ages, reionize the universe, and provide the enriched gas required for later stellar generations. They may also be important as precursors to black holes that coalesce and power bright early quasars. The first stars are thought to form inside haloes of dark matter of mass $10^5 - 10^6 M_\odot$ at redshifts $z = 10 - 50$. These haloes arose from the merging of smaller structures, as overdense regions in the universe assemble heirarchically into ever larger haloes. The haloes consist of 85% dark matter and 15% baryons in the form of pristine hydrogen and helium (from Big Bang nucleosynthesis). The baryonic matter cools and collapses via molecular hydrogen cooling [11] into a single small protostar [13] at the center of the halo (for reviews see e.g. [1, 2, 3]).

In this paper we consider the effect of the dark matter particles on the formation process of these first stars. We focus on the most compelling dark matter candidate, the lightest supersymmetric particle. Supersymmetry (SUSY), at this point a beautiful theoretical construct, has the capability of addressing many unanswered questions in particle theory as well as providing the underpinnings of a more fundamental theory such as string theory. If SUSY is right, then for every known particle in the universe, there is an as yet undiscovered partner. The lightest of these, the Lightest Supersymmetric Particle or LSP, would provide the dark matter in the universe. The search for SUSY is one of the motivations for building the Large Hadron Collider at CERN, and one may hope that it will be discovered as early as 2008.

The LSP is the favorite dark matter candidate of many physicists. This is true not only because of the beautiful properties of SUSY, but also because the LSP automatically has the right properties to provide 24% of the energy density of the universe. In particular, the neutralino, the SUSY partner of the W, Z, and Higgs bosons, has the required weak interaction cross section and $\sim$ GeV - TeV mass to give the correct amount of dark matter in the universe today. The SUSY particles are in

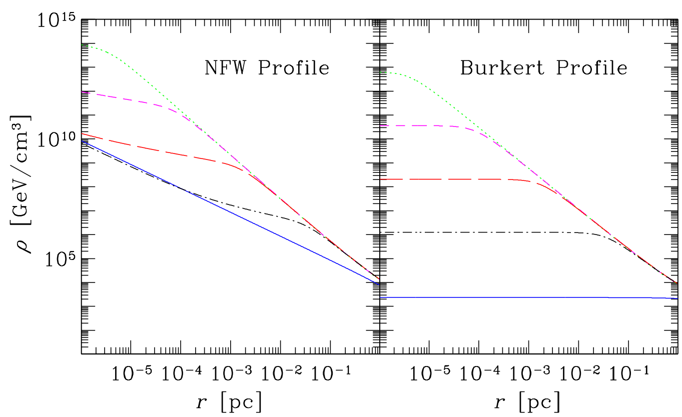

FIG. 1: Adiabatically contracted DM profiles for (a) an initial NFW profile and (b) an initial Burkert profile, for $M_{\rm vir} = 10^6 M_\odot$, $c_{\rm vir} = 10$, and $z = 19$. The blue (solid) lines show the initial profile. Black (dot-dash) lines correspond to a baryonic core density of $10^7 \rm cm^{-3}$, red (long-dashed) lines to $10^{10} \rm cm^{-3}$, magenta (dashed) lines to $10^{13} \rm cm^{-3}$, and green (dotted) lines to $n \sim 10^{16} \rm cm^{-3}$.

thermal equilibrium in the early universe, and annihilate among themselves to produce the relic density today. It is this same annihilation process that is the basis of the work we consider here. The SUSY particles, also known as WIMPs (Weakly Interacting Massive Particles), annihilate with one another wherever their density is high enough. Such high densities are achieved in the early universe, in galactic haloes today [4, 5], in the Sun [6] and Earth [7, 8], and, as we will show, also in the first stars. As our canonical values, we will use the standard value $\langle \sigma v \rangle = 3 \times 10^{-26} \rm cm^3/sec$ for the annihilation cross section and $m_\chi = 100$ GeV for the SUSY particle mass; but will also consider a broader range of WIMP masses (1 GeV–10 TeV) and cross-sections [35]. The effects we find apply equally well to other WIMP candidates, such as sterile neutrinos or Kaluza-Klein particles.

DM density profile. In order to study the effects of DM on the star formation process, we need to know its density

profile inside the baryonic core. While simulations have obtained remarkably good density profiles for the collapsing protostellar gas, they have unfortunately (as yet) been unable to do so for the DM. Thus we use adiabatic contraction [15] to obtain estimates of the DM profile. As our initial conditions, we take an overdense region of $10^5 - 10^6 M_\odot$ with an NFW profile [14] for both DM and gas, where the gas contribution is 15% of that of the DM. (For comparison, we also use a Burkert profile [18], which has a DM core before contraction.) Then, as the gas collapses, we allow the DM to respond to the changing baryonic gravitational potential (gas density profiles taken from simulations of [16, 17]). The final DM density profiles are computed with adiabatic contraction ($M(r)r$ = constant) and are shown in Fig. 1 for concentration parameter $c = 10$ at a redshift $z = 19$ and halo mass $M = 10^6 M_\odot$. It is important to point out that our results do not change much for other choices of these parameters; e.g. even for $c = 1$, the dark matter density only changes by a factor of 4. After contraction, the DM density at the outer edge of the baryonic core is roughly $\rho_\chi \simeq 5\mathrm{GeV/cm}^{-3}(n/\mathrm{cm}^3)^{0.81}$ and scales as $\rho_\chi \propto r^{-1.9}$ outside the core.

Our adiabatically contracted NFW profiles match the DM profile obtained numerically in [16] (see their Fig.2). They present two DM profiles (their earliest and latest profiles), for $n \sim 10^3\mathrm{cm}^{-3}$ and $n \sim 10^{13}\mathrm{cm}^{-3}$ as far inward as 5×10^{-3}pc and 0.1pc. The slope of these two curves is the same as ours. $\rho_\chi \propto r^{-1.9}$. If one uses this slope and extrapolates inward to smaller radii and to higher densities, then one obtains the same DM densities as with our adiabatic contraction approach. We are encouraged by this agreement. It is interesting to note that adiabatically contracted objects of any mass, even outside of the context of Pop III stars, might lead to enhanced DM annihilation leading to photons observable by GLAST or to a modified reionization history.

As a caveat, we note that the approach of adiabatic contraction must be used with caution. It formally requires the orbital particle time to be short compared to the collapse time, though in practice the method works well beyond this limit (see e.g. [19]). We are also concerned about the requirement of spherical symmetry, when in fact there are filaments, clumps, and mergers, so that dynamical friction or violent relaxation may take place. The gas may form bars that could sweep out DM, though the rotation timescale for bars may be too long for them to be important. We encourage simulators to improve the DM resolution in first stars to confirm our results. The closest previous work is that of Merritt [20], who used initial profiles $\rho \propto r^{-\gamma}$ with $\gamma = [0, 2]$ around a central black hole and found final profiles $\rho \propto r^{-\gamma'}$ with $\gamma' = [2.25, 2.5]$ (i.e. even steeper than we what we found). We caution the reader that our heating estimates below rely upon DM densities obtained with these assumptions.

DM Heating. WIMP annihilation produces energy at a rate per unit volume $Q_{\rm ann} = \langle\sigma v\rangle\rho_\chi^2/m_\chi \simeq 1.2 \times 10^{-29}\mathrm{erg/cm^3/s}\ (\langle\sigma v\rangle/(3 \times 10^{-26}\mathrm{cm^3/s}))\ (n/\mathrm{cm}^{-3})^{1.62}(m_\chi/(100\mathrm{GeV}))^{-1}$. In the early stages of Pop III star formation, when the gas density is low, most of this energy is radiated away. However, as the gas collapses and its density increases, a substantial fraction f_Q of the annihilation energy is deposited into the gas, heating it up at a rate $f_Q Q_{\rm ann}$ per unit volume.

Previous work [21, 22] considered the effects of DM annihilation on earlier low density phases of Pop III star formation ($n \lesssim 10^4\mathrm{cm}^{-3}$). They rightly concluded that 100 GeV neutralinos cannot heat the protostar at these low densities because their annihilation products simply escape out of the object without depositing much energy inside. They consequently focused on lighter particles, such as 1–10 keV sterile neutrinos and 1–100 MeV light dark matter.

However, we find that, for WIMP mass $m_\chi = 100$GeV (1 GeV), a crucial transition takes place when the gas density reaches $n > 10^{13}\mathrm{cm}^{-3}$ ($n > 10^9\mathrm{cm}^{-3}$). Above this density, most of the annihilation energy remains inside the core and heats it up to the point where further collapse of the core becomes difficult. At this point the DM density is 2% (10%) of the gas density in the core, the size of the core is ~ 17 A.U. (~ 960 A.U.), its mass is $\sim 0.6 M_\odot$ ($\sim 11 M_\odot$), $f_Q \sim 2/3$, and the energy produced by DM heating is $\sim 140 L_\odot (\sim 835 L_\odot)$.

Here we estimate the fraction f_Q of DM annihilation energy that remains inside the gas core (more detailed work is in progress). f_Q scales linearly with the gas density and depends on the relative number of the various annihilation products and their energy spectrum. The latter are heavily dependent on the WIMP model. From our experience with neutralino DM, we assume the following typical values: about 30% of the energy goes into neutrinos, 30% into photons, and 30% into stable charged particles like electrons and positrons. Unstable particles like neutral pions, charged pions, and muons decay into photons, neutrinos, and electrons before exiting the cloud. The energy spectrum of photons and electrons depends to some extent on the exact annihilation channels. For our purpose, we consider typical spectra produced in Pythia simulations of 500 GeV neutralino annihilation [23, 24]. Other spectral shapes will change the precise values of our results but not the overall effect.

Neutrinos escape from the cloud without depositing an appreciable amount of energy. Electrons below a critical energy $E_c \approx 280$ MeV in hydrogen lose energy predominantly by ionization. Higher energy electrons do so by emission of bremsstrahlung photons. As these bremsstrahlung photons pass near gas nuclei, they create electrons and positrons, which in turn may generate other bremsstrahlung photons. Thus starts a sequence of photon, electron, and positron conversions: an electromagnetic (EM) cascade. While photons above ≈ 100 MeV also initiate an EM cascade, lower energy photons trans-

fer most of their energy to electrons in the gas (Compton scattering).

We approximate the energy loss of electrons via ionization with $4.41\mathrm{MeV}/E$ $(\mathrm{g/cm^2})^{-1}$. For EM cascades, we assume a gamma distribution for the mean longitudinal profile. Thus the fraction of energy lost in traversing a thickness X of gas equals $\gamma(a, bX/X_0)/\Gamma(a)$, where $\gamma(x, y)$ is the incomplete gamma function, $X_0 = 63$ g/cm^2 is the radiation length in hydrogen, $a = 1 + b[\ln(E/E_c) - 0.5]$, and $b = 0.5$ [26]. We estimate the core thickness as $X = 1.2 m_p n r_0$. Here m_p is the proton mass, r_0 is the core radius, and the factor of 1.2 is appropriate for a uniform sphere. We model the fraction of energy loss of photons by converting each photon to an electron of the same energy after one photon attenuation length. The latter is computed from formulas in [27], interpolated to produce the hydrogen curve in [26], figure 27.16.

Annihilation of DM outside the core can also contribute to heating inside the core. The annihilation products are not stopped until they reach the core because the baryon density outside falls as $r^{-2.3}$. We integrate the effects of DM particles annihilating in the external region at $r > r_0$, whose annihilation products have a probability of hitting the core given by $\pi r_0^2/4\pi r^2$. Taking $\rho_\chi \sim 1/r^2$ outside the core, we find a 25% enhancement in core heating due to the external region. Henceforth our results are obtained without this enhancement factor.

Results. To compare with DM heating, we include all relevant cooling mechanisms. The dominant mechanism is H_2 cooling (we use the rates in [11]), though our code also includes other effects such as H line cooling (important only at high temperatures) and Compton cooling (important only if the gas becomes very ionized) [28]. We use the opacities from [29]; e.g. at $n \sim 10^{13}\mathrm{cm}^{-3}$, we take a $\sim 8\%$ cooling efficiency. Setting the heating rate equal to the cooling rate gives the critical temperature $T_c(n)$ at a given density n below which heating dominates.

In figure 2 we compare $T_c(n)$ to typical evolution tracks in the temperature-density phase plane. The blue (solid) and green (dotted) lines show the temperature evolution of the protostellar gas in the simulations (without DM) of [29] and [17] respectively. The red (dashed and dot-dashed) lines show the critical temperature: below these lines, DM heating dominates over all cooling mechanisms.

Figure 2 illustrates results for a range of WIMP masses from 1 GeV–10 TeV for a canonical $3 \times 10^{-26}\mathrm{cm}^3/\mathrm{sec}$ annihilation cross-section. Since the heating rate scales as $\langle\sigma v\rangle/m_\chi$, these same curves equivalently apply to a variety of cross-sections for a given WIMP mass.

The important result is that the blue/green (evolutionary) and red (critical temperature) lines always cross, regardless of WIMP mass or H_2 fraction: this is a robust result. As soon as the core density reaches this crossing point, the DM heating dominates inside the core and changes its evolution. Notice that at m_χ =1 GeV, the crossing point for small H_2 fraction is at low densities, around $n \sim 10^5\mathrm{cm}^{-3}$, in agreement with [21]. If the H_2 fraction is increased, cooling dominates for a longer time, as expected, but not forever.

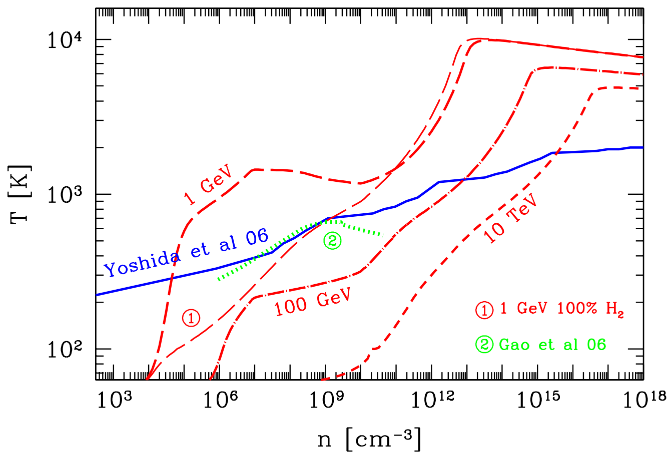

FIG. 2: Comparison of critical temperature (red dashed lines) to typical evolution tracks in the temperature-density phase plane. The blue (solid) and green (dotted) lines show the protostellar gas evolution from simulations of [29] and [17] respectively. The red dashed lines mark $T_c(n)$ for: (i) $m_\chi = 1$ GeV with H_2 density from simulations, (ii) $m_\chi = 1$ GeV assuming 100% H_2, (iii) $m_\chi = 100$ GeV and (iv) $m_\chi = 10$ TeV. At the crossing point of the blue/green and red lines, DM heating dominates over cooling in the core's evolution.

Our results were obtained for two possible values for the H_2 fraction: the value given by the simulations without DM, and the case of 100% molecular hydrogen. The fully molecular case produces the most effective cooling of the core and hence may underestimate the importance of DM heating. DM annihilation gives rise to additional electrons which could in principle produce more H_2 and go in the direction of cooling the star; however, for 100 GeV mass WIMPs, the DM only becomes important at such high gas densities that the core has already reached a 100% H_2 fraction and this effect is irrelevant. For lighter WIMPs, the transition to DM heating may occur at a lower H_2 fraction and a different baryon density.

The crossing point is stable against temperature changes at constant density and composition: if the temperature increases away from the critical value, cooling begins to dominate and drives the temperature back down to the critical value; if the temperature decreases, heating dominates and lowers the temperature. The reason for this behavior is that the H_2 cooling rate increases with temperature while the DM heating is independent of temperature.

As soon as the DM annihilation products are contained inside the protostellar core, the heating dominates over the cooling. Hence, for 100 GeV (1 GeV) neutralino DM, once the gas density reaches a critical value

of $\sim 10^{13}\text{cm}^{-3}$ (10^{9}cm^{-3}), the heating rate from DM annihilation exceeds the rate of hydrogen cooling. The protostellar core is *prevented from cooling and collapsing* further.

Discussion. Our main conclusion is that the standard picture of Pop III star formation is drastically modified by neutralino dark matter annihilation inside the protostellar object. The DM annihilation provides a heat source that exceeds any cooling mechanism and thereby hinders the further collapse of the protostar.

We propose that a new type of object is created, a "dark star" supported by DM annihilation rather than fusion. One question is how long such a phase of stellar evolution lasts. If such an object were stable for a long time period, it would even be possible for these dark stars to still exist today. Dark stars could last as long as the DM annihilation timescale, $\tau_e = m_\chi/(\rho_\chi\langle\sigma v\rangle) \simeq 0.6\,\text{Gyr}\,(n/10^{13}\text{cm}^{-3})^{-0.8}\,(m_\chi/100\text{GeV})\,(\sigma v/3\times10^{-26}\text{cm}^3\text{s}^{-1})^{-1}$. For our canonical case, we find $\tau_e \sim 600$ Myr (~ 15 Myr) for $n = 10^{13}\text{cm}^{-3}$ ($n = 10^{15}\text{cm}^{-3}$). By comparison, the entire timescale for collapse (without taking into account DM annihilation) is ~ 1 Myr at $z = 50$ or 100 Myr at $z = 15$; even for the more recent episodes, the dynamical time at the high densities considered here is very short ($< 10^3$ yr). However, after this DM annihilates away, it is possible that the DM hole in the small central core can fill in again, depending on the DM orbits at this stage. DM further out can also continue to heat the core. On the other hand, as baryons continue to accrete onto the protostar, it is possible that the annihilation shuts down sooner. The lifetime of the dark star phase is crucial to addressing the question of the effects it has on the universe.

Much further work is required to address the evolution of these objects; we here briefly speculate as to possible outcomes. One possibility is that the dark star phase still persists today, so that these objects never reach the main sequence. A second possibility would be a shorter dark star phase, during which the gas core is in a state of quasi-hydrostatic equilibrium. For our 100 GeV case, both the surface gravity and the hydrostatic force per unit mass from the temperature and pressure gradient are of the same order of magnitude, $\sim 10^{-3}$ cm/s^2. Outer material would continue to accrete onto the quasi-hydrostatic core [31], probably accompanied by the formation of an accretion shock. Alternatively, once the core's central temperature reaches $\sim 10^6$ K, deuterium burning and a pp chain may start. The star would then emit radiation and contract in a Kelvin-Helmholtz timescale, until the central density and pressure is high enough to start the CNO cycle. The star would finally reach the zero-age main sequence. In this scenario, Pop III star formation would be delayed rather than blocked. A third possibility would be that the core's contraction slows down as a consequence of DM heating, and yet the core continues to contract further. Then DM heating would continue to dominate over available cooling mechanisms at ever higher baryon densities.

The effects of a dark star phase of stellar evolution could be very interesting. The reionization of the IGM could be quite different, as would be the production of the heavy elements required to form all future generations of stars. We clearly need to recompute stellar structure with this new heat source, to see how different dark stars would look relative to the ordinary fusion-driven stars. It is possible, e.g., that dark stars are luminous but at lower temperatures, so that UV radiation does reionize the IGM yet the scenario is quite different. It is possible, e.g., that reionization is delayed. Perhaps the discrepancy between measurements of σ_8 by WMAP3 and in Lyman-α could be resolved in this way.

DM heating may also alter the mass of Pop III stars. Even without DM heating, the mass is still uncertain. Refs. [17, 32] have explored different possibilities for the accretion process (disk vs. spherical) of baryonic matter onto the central protostar as well as sensitivity to cosmology and found great uncertainty as to the final stellar mass. DM heating could also affect the result. For example, in the case of spherical accretion, the heating might produce radiation at the Eddington luminosity whose pressure prevents further accretion. Thus, the DM heating might lead to lower mass stars than in the standard picture. Alternatively the initial protostellar object may be larger, and dark stars might accrete enough material [30] to form large black holes [33, 34] en route to building the (as yet unexplained) $10^9 M_\odot$ black holes observed at $z \sim 6$.

What are other observational consequences of a "dark star"? If these objects are luminous but differ from ordinary stars (e.g. shine at lower temperatures), then James Webb Space Telescope could in principle find them (at z$\sim$10) and differentiate their spectra from those expected in the standard first star formation scenarios. In addition, one might hope to detect the DM annihilation products such as neutrinos and γ-rays. However, the angular resolution of current and planned detectors (AMANDA, ICECUBE, GLAST, HESS, VERITAS, MAGIC) is not good enough to identify an individual dark star source at $z > 10$, so that the ν and γs would add to the extragalactic backgrounds and could provide limits at best. Remnants today of the dark star phase would be more testable, such as stellar remnants still in enhanced DM distributions. Today's remnant 10^6 DM halos that were once the site of star formation may be modified due to the adiabatic contraction that took place earlier, so that enhanced DM annihilation might still occur today and these objects could be identified in upcoming experiments as individual ν or γ sources.

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[35] The details of the interactions and masses of the neutralinos depend on a large number of model parameters. In the minimal supergravity model, experimental and observational bounds restrict the neutralino mass m_χ to 50 GeV–2 TeV, while the annihilation cross section σv lies within an order of magnitude of $\langle\sigma v\rangle = 3\times10^{-26}\mathrm{cm}^3/\mathrm{sec}$ (except at the low end of the mass range where it could be several orders of magnitude smaller). Nonthermal particles can have annihilation cross-sections that are many orders of magnitude larger (e.g. [12]) and would have even more drastic effects.